FIRST REPORT

On

ELECTRICITY AND MAGNETISM

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Construction and use of an electric balance based on the property of metal threads of having a torsional restoring force proportional to the angle of torsion.

Experimental determination of the law according to which the parts of bodies charged with the same type of electricity repel one another.

In a report presented to the Academy in 1784 I determined by experiment the force law for torsion of a metal thread, and I found that this force was proportional to the torsion angle, to the fourth power of the diameter of the suspension thread, and to the inverse of its length, all multiplied by a constant coefficient that depends on the nature of the metal, and that is easy to determine by experiment.

I demonstrated in this report that by means of this torsional force it was possible to measure with precision very weak forces, for example one ten thousandth of a grain [$\sim 5\mu g$; 1 grain = 0.053 grams]. In the same report I gave a first application of this theory by trying to evaluate the force constant assigned to adhesion in the formula that expresses friction of the surface of a solid body moving in a fluid.

Today I am showing the Academy an electrical balance built according to the same principles; it measures with the greatest exactitude the state and the electric force of a body, however feeble the charge.

Construction of the Balance

Although practice had taught me that carrying out multiple electrical experiments in a convenient manner requires correcting several faults in the first balance of this type that I have had built; still, since it is the only one available to me thus far, I will describe it while noting that its shape and size could and should be altered according to the sort of experiment one intends to do. The first figure shows in perspective this balance, which is detailed below.

On a glass cylinder ABCD, 12 inches in diameter and 12 inches in height, one places a glass plate 31 inches in diameter, covering the whole vessel with glass; this plate has two holes about 20 lignes [1 ligne = 1/12 inch = 2.25 mm] in diameter, one in the middle, at *f*, on which a glass tube 25 inches tall is mounted; it is cemented on the hole *f* with the cement used in electrical apparatus : at the upper end of the tube at *h*, is placed a torsion micrometer shown in detail in *figure 2*. The top piece, *number I*, has the knob *b*, the pointer *io*, and a suspension clamp *q*; this part is inserted in the hole *G* of part *number 2*; this part *number 2* consists of a circle *ab*, whose rim is divided into 360°, and a copper tube ϕ inserted into tube *H*, *number 3*, which is glued inside the top of the vertical glass tube *fh* of *figure 1*. Clamp *q*,

figure 2, number 1, is shaped rather like the end of a porte-crayon [an early mechanical pencil] that can be tightened by means of ring q; this clamp holds the end of a very fine silver thread; the other end of the silver thread is held (*fig. 3*) at P by the clamp of a copper or iron cylinder Po whose diameter is barely one ligne and whose end P is split forming a clamp closed by means of slider ϕ . This little cylinder is widened and pierced at C, to allow sliding in the needle ag (*fig. 1*) : this little cylinder must be heavy enough to stretch the silver thread taut without breaking it. The needle shown as ag (*fig. 1*), suspended horizontally about half-way up the large container that surrounds it, is made of either a silk thread stiffened by Spanish wax or a piece of straw similarly stiffened by Spanish wax ending with a shellac cylinder 18 lignes long from q to a : on end a of this needle is a little elderberry pith-ball two or three lignes in diameter ; at g is a little vertical leaf of paper moistened with turpentine [?] serving as a counter weight to ball a and slowing the oscillations.

As we said, cover AC contained a second hole at m; into this second hole one inserts a small cylinder $m\phi t$ whose lower portion ϕt is made of shellac; at t is another pith-ball; around the container, at the height of the needle one makes a circle zQ divided into 360° : for greater simplicity I used a strip of paper divided into 360° , which I glued around the container at the height of the needle.

To begin using this instrument I place the cover so as approximately to make the hole m at the first division, or at point o of the circle zoq traced out on the container. I place the pointer oi of the micrometer at point o or the first division of the micrometer; I then rotate the entire micrometer in the vertical tube fh until, sighting by the vertical thread suspending the needle and the center of the ball, the needle ag is aligned with the first division of circle zoq. I then introduce through hole m the other ball t suspended by the rod $m\phi t$, so that it touches ball a, and sighting by the center of the suspension thread and ball t one sees the first division o of circle zoq. Now the balance is ready for all the operations; we will give as an example how we used it to determine the fundamental law according to which electrified bodies repel one another.

The Fundamental Law of Electricity

The repulsive force between two small spheres charged with the same type of electricity is inversely proportional to the square of the distance between the centers of the two spheres.

EXPERIMENT

One charges a small conductor, *fig.* 4, that is nothing other than a needle with a large head, insulated by sticking its point into the end of a rod of Spanish wax; one sticks this needle into hole m, and touches ball t, in contact with ball a: removing the needle the two balls are charged with the same type of electricity, and undergo mutual repulsion to a distance that one measures by sighting across the suspension thread and the center of ball a to the corresponding division of circle zoq: turning the pointer of the micrometer in the direction *pno* one twists the suspension thread lp, and produces a force proportional to the angle of torsion, which tends to draw ball a toward ball t. By this means one observes the distance to which various angles of torsion bring ball a toward ball t, and by comparing the

torsional forces with the corresponding distances between the two balls one determines the repulsion law.

Here I shall present only a few trials that are easy to repeat and that immediately show the repulsion law.

First Trial. Having charged the two balls with the head of the pin, the micrometer pointing to o, ball a of the needle moved 36° from ball t.

Second Trial. Having twisted the suspension thread by means of knob o of the micrometer by 126°, the two balls drew closer and stopped at 18° of separation from one another.

Third Trial. Having twisted the suspension thread by 567°, the two balls drew closer to 8 and a half degrees.

Explanation and result of this experiment.

Before the balls are charged they touch one another, and the center of ball *a*, suspended on the needle is displaced from the point where the torsion of the suspension thread is null only by half the diameters of the two balls. It is important to note that the silver thread *lp*, which forms the suspension, was 28 inches long and so thin that a foot's length weighed only 1/16 grain [*i.e.* diameter about 1/5 that of a human hair]. Calculating the force required to twist this thread, acting at point *a* four inches from thread *lp* or the center of suspension, I found by the formulae explained in a paper on the law of torsional force for metal threads, printed in the *volume of the Academy for 1784*, that to twist this thread by 360° it was only necessary at point *a*, acting with the lever arm *an*, to use a force of 1/340 grains : thus, since torsion forces are, as proved in this paper, proportional to the torsional angle, the least repulsive force between the two balls would distance them one from the other.

We find in our first experiment, where the micrometer pointer is on point *o*, that the balls are 36° apart, producing at the same time a torsional force of $36^\circ = 1/3400$ grains. In the second trial the distance between the balls is 18° , but since one has twisted the micrometer by 126° , the repulsive force was 144° : thus at half the first distance the repulsion between the balls is quadrupled.

In the third trial we twisted the suspension thread by 567° and the two balls are only 8 and a half degrees apart. The total torsion was thus 576°, quadruple that of the second trial, and there lacked only a half degree in the distance between the two balls for the third trial to be reduced to half that of the second. It results thus from these three trials that the repulsion that two balls charged with the same kind of electricity exert on one another is in inverse proportion to the square of the distances.

First Remark

On repeating the preceding experiment one will observe that using a silver thread as fine as the one we used, which for an angle of 5° gives a force of only approximately a 24 thousandth of a grain, however still the air, and whatever care one takes, one can only measure the natural position of the needle, without torsion, to within 2 to 3°. Thus in order to have a first experiment to compare with the following ones, it is necessary, after having charged the balls, to twist the suspension thread by 30 to 40°, which together with the observed distance between the two balls, will give a torsional force large enough that the 2

or 3° uncertainty in the initial position of the needle, when the torsion is null, would not generate a measurable error in the results. It is also necessary to be aware that the silver thread that I used in this experiment is so fine that it breaks with the least tangling : I subsequently found that it was more convenient to use a suspension thread of nearly twice the diameter for these experiments, although its torsional flexibility was fourteen to fifteen times smaller than that of the first. Before using this silver thread, it is necessary to be careful to stretch it for two or three days with a weight that is about half what it can support without breaking ; It is again necessary to note that using this latter silver threat one must never twist it beyond 300°, because beyond this torsional limit it begins to deform and recovers, as we proved in the paper cited above, printed in 1784, only with forces smaller that this torsional angle.

Second Remark

The charge of the two balls decreases a bit during the course of the experiment ; I found that on the day when I made the preceding trials the charged balls, being repelled to 30° from one another with a torsional angle of 50° , drew nearer to one another by 1° over three minutes; but since I needed only two minutes to make the three preceding trials, one can in these experiments neglect the error resulting from loss of charge. If one desires a greater precision, or when the air is humid and charge is lost quickly, one should as a preliminary observation determine the amount of diminution of electrical action of the two balls in each minute and then use this preliminary observation to correct the results of the experiments one would wish to do on that day.

Third Remark

The distance between the two balls, when they are separated from one another by their reciprocal repulsion, is not precisely measured by the angle between them, but by the chord of an arc that joins their centers ; just as the lever at the end of which the force is applied is not measured by half the length of the needle, or by the ray, but by the cosine of half the angle formed by the distance of the two balls; these two quantities, of which one is smaller than the arc and thus diminishes the distance measured by this arc, while the other diminishes the lever, compensate in a certain fashion; and in experiments of the type of those we are involved in one can without significant error stick with the evaluation that we have given, if the distance between the two balls is less than 25 to 30°; in other cases it is necessary to make the rigorous calculation.

Fourth Remark

As the experiment proves that in a well closed chamber one can determine the position of the needle with the first silver thread to within 2 or 3° when the torsion is null, yielding, according to the proportionality of torsional force to torsional angle, forces no larger than a forty-thousandth of a grain, the weakest amounts of electrification can be measured easily with this balance. For this operation one pushes , *fig 5*, through a plug of Spanish wax a little copper rod *cd*, ending at *c* in a crook, and at *d* in a little gilded pith-ball, and one puts the plug *A* into the hole *m* of the balance *fig. 1*, so that the center of the ball *d*, viewed across

the suspension thread corresponds to point o of the circle zoq; then putting an electrified body near the crook c, however feeble the electrification of the body, ball a separating from ball d indicates the electrification, and the distance between the two balls measures the force according to the principle of inverse proportion to the square of the distance.

But I must mention that since these first experiments I have had built different small electrometers according to the same principles of torsional force, using for the suspension thread a silk filament such as it comes from the cocoon, or the hair from an Angora goat. One of these electrometers of nearly the same form as the electrical balance described in this paper is much smaller ; it is only 5 to 6 inches in diameter, with a one inch column; the needle is a little thread of shellac 12 lignes long ending at *a* in a very light circle of gold leaf. The needle and the gold leaf weigh about a quarter of a grain; the suspension thread, as it comes from the cocoon, being 4 inches long, has such a flexibility that with a lever arm of one inch only a sixty-thousandth of a grain is necessary to twist a whole circle or 360°: presenting the crook C of this electrometer figure 5 with an ordinary stick of Spanish wax, electrified by rubbing, 3 feet away from the crook the needle is repelled to more than 90°. Later we will describe this electrometer in detail when we wish to determine the nature and the degree of electrification of different substances that acquire a weak charge by rubbing one against the other.