## Chemistry 125 Second Examination Answers

## October 17, 2005

The exam budgets 50 minutes, but you may have 60 minutes to finish it. Good ar swers can fit in the space provided.

1. These two diagrams illustrate the HOMO and LUMO of an uncharged molecule $\mathrm{H}_{2} \mathrm{XYH}_{2}$, where the four lighter balls are H , and the two darker balls are X and Y , atoms from the second row of the periodic table.
(Relative wavefunction signs should be obvious.)
A) $(2 \mathrm{~min})$ Under each diagram write the orbital's name (HOMO or LUMO). AND name the single H-like "united-atom" orbital to which it is "plum-pudding" analogous.

[N.B. these are $\pi^{*}$ and $\pi$ MOs, but the question asks for analogous AO types]



HOMO
$2 p$
B) (3 min) Identify the atoms $\mathbf{X}$ and $\mathbf{Y}$ as specific second row elements, AND explain the shapes of these orbitals.

X and Y obviously have different nuclear charge to give unsymmetrical $\pi / \pi$ * MOs. X must have the higher nuclear charge, since it dominates in the more favorable $\pi$ HOMO, which has one node, rather than two. Since the molecule is uncharged, the X and $Y$ must be $\mathbf{N}$ and $\mathbf{B}$, respectively.
C) $(2 \mathrm{~min})$ Draw two reasonable resonance structures for $\mathrm{H}_{2} \mathrm{XYH}_{2}$ (relabel X and Y as in your answer to $\mathbf{B}$ )

[N.B. tetravalent N is positive; tetravalent B , negative]
D) ( 1.5 min ) On the figure at the top draw an arrow to show the direction from which you would expect hydroxide to attack the LUMO AND write a few words to explain your choice of direction .

To maximize overlap the HOMO of $\mathrm{OH}^{-}$approaches the large lobe of the LUMO from a direction as far as possible from the LUMO's nodes.
E) ( 3.5 min ) Draw structures with curved arrows to explain how attack by hydroxide would influence the XY distance.

2. ( 3.5 min ) Mention two ways Chladni figures are relevant to quantum mechanics.

1) They illustrate the concept of different nodes (linear and circular in 2D).
2) Increasing numbers of nodes give increasing frequency (energy).
3) They illustrate degeneracy (same frequency for different combinations of nodes).
4) They were explained using the same kind of mathematics subsequently used to solve Schrödinger's equation for 1 -electron atoms.
3. (4 minutes) Briefly explain the relationship between SCF calculations and correlation energy.

SCF calculations attempt to treat e-e repulsion in many-electron systems by holding all electrons but one in fixed average clouds for purposes of calculate the potential in which the one electron moves. This is erroneous because the other electrons can shift dynamically to avoid close approach to the electron in question. Thus SCF must overestimate e-e repulsion. This error is called "correlation energy".
4. (6 minutes) Draw an orbital-mixing energy diagram to explain how $\mathrm{BH}_{2}-\mathrm{CH}=\mathrm{CH}_{2}$ might differ in reactivity from $\mathrm{BH}_{2}-\mathrm{CH}_{2}-\mathrm{CH}_{3}$.

$$
\mathrm{BH}_{2}-\mathrm{CH}_{2}-\mathrm{CH}_{3} \quad \mathrm{BH}_{2}-\mathrm{CH}=\mathrm{CH}_{2}
$$

## $\sigma * C-C$



In the ethyl $\left(\mathrm{CH}_{2} \mathrm{CH}_{3}\right)$ case on the left there is little mixing between the unusually low
LUMO on B and the $\sigma$ HOMO because of poor E-match. In the vinyl $\left(\mathrm{CH}=\mathrm{CH}_{2}\right)$ case on the right there is substantial mixing with the $\pi \mathrm{HOMO}$, so that both LUMO and HOMO become less reactive. This is a case of important resonance stabilization.
[Note the analogy with the relatively unreactive amide group.]
5. Consider the coordinate system shown, where a typical point at $(x, y, z)$ is at a distance $r_{\mathrm{F}}$ from the origin and at a distance $r_{\mathrm{H}}$ from a fixed point on the x axis.


That is, where would it appear in a quantum
a) $(9.5 \mathrm{~min})$ Explain why one might be interested in the value of the following function:

$$
x e^{-\left(k_{1} r_{F}+k_{2} r_{H}\right)}
$$ mechanical calculation, and what would be its significance?

[Remember: $e^{-\left(k_{1} r_{F}+k_{2} r_{H}\right)}=e^{-k_{1} r_{F}} e^{-k_{2} r_{H}}$ ]
The function in question is a PRODUCT of $x e^{-k_{1} r_{F}}$ and $e^{-k_{2} r_{H}}$. The first is a $2 p_{x}$ orbital centered at the origin (F). The second is a $1 s$ orbital centered on the fixed point to the right (H).

Such a product arises in the " $2 A B$ " cross-term from squaring the LCAO molecular orbital $\left(2 p_{\mathrm{xF}}+1 s_{\mathrm{H}}\right)$, the $\sigma$ bond of the HF molecule. This is the "overlap" function that gives rise to increased electron density between nuclei resulting in bonding.
[Product functions also occur in two- (or many-) electron wave function, but this function is of only $x, y, z$, the coordinates of a single electron. Thus it is an LCAO overlap function, not a two-electron function.]
b) ( 3 min ) In the formula above, what might make $k_{1}$ different from $k_{2}$ ?

These constants determine how rapidly the exponentials decay upon moving away from the nuclei. The constant for F should be greater than that for H , because the higher nuclear charge holds the electrons closer to the nucleus.
[In fact, $\mathrm{k}_{1}$ should be $9 / 2 \times \mathrm{k}_{2}, 9$ for the 9 -fold greater nuclear charge, but divided by 2 because of the increased principal quantum number for $2 p$ vs 1 s . This is the scaling of $\rho$ vs $r$ ].
6. (12 minutes) In the infrared spectrum of interstellar clouds there is a band at $\mathbf{9 0 3 . 3 9} \mathbf{~ c m}^{-1}$ which astronomers have recently assigned to the out-of-plane bending vibration of the ion $\mathbf{N H}_{3}{ }^{+}$(ammonia which has lost one electron).

Reasoning from what you know of $\mathrm{XH}_{3}$ molecules, explain whether $\mathrm{NH}_{3}{ }^{+}$at its lowest potential energy should be planar or pyramidal AND rationalize the $903 \mathbf{~ c m}^{-1}$ frequency.
$\mathrm{NH}_{3}{ }^{+}$has the same number of valence electrons as $\mathrm{CH}_{3}$, so the argument that three $\mathrm{N}-\mathrm{H}$ bonds will be strongest with $\mathrm{sp}^{2}$ hybridization, while the central atom with four singly occupied orbitals doesn't care ("whatever") about hybridization would seem to be the same predicting a single minimum potential for out-of-plane bending.

There is a difference, however, which I didn't really expect anyone to see.

The extra proton in the N nucleus causes the N-H bonding electron pairs to be held closer to the N than to the H nucleus. Thus from the point of view of the N atom, there is more than one electron in each bonding AO , but only one electron in the unshared orbital. [This reverses the effect that makes $\mathrm{NH}_{3}$ pyramidal because there is one electron in each bonding AO, but two electrons in the unshared AO.]

This means that $\mathrm{NH}_{3}{ }^{+}$has more reason to be flat, and more resistance to bending, than does $\mathrm{CH}_{3}$. Since the vibration of $\mathrm{CH}_{3}$ comes at $606 \mathrm{~cm}^{-1}$, one expects a higher vibration frequency $\left(903 \mathrm{~cm}^{-1}\right)$ in the ammonia cation case.

